Valentyn Khokhlov, CFA, PhD in Technical Sciences
International Marketing Manager, Global Spirits, Ukraine

INCORPORATING HUMAN CAPITAL INTO THE ASSET ALLOCATION

In this paper we introduce a framework for incorporation human capital into the assets allocation in financial planning and research the effect of correlation between the labor income and the stock market returns on choosing the optimal portfolio for individual investors. While we found the strong evidence that labor income is correlated with the past stock market return, its actual effect on asset allocation becomes significant only if the standard deviation of labor income is comparable to the standard deviation of the stock market returns. The Monte Carlo simulation shows that incorporating human capital into asset allocation leads to significantly different portfolio structure, however adding the correlation between the labor income and stocks into mean-variance optimization does not lead to increase in the accumulated wealth.

Key words: human capital, labor income, asset allocation, financial planning.

Introduction

Traditional asset allocation problem is to determine weights of different classes of financial assets (such as stocks and bonds) in the client's portfolio in order to maximize the client's wealth. When applied to individuals, this definition of the problem has one significant drawback — most part of the total wealth an average individual earns during her lifetime is not generated by financial assets. According to Chen[1], the largest part of one's wealth is typically generated by the labor income. Ignoring human capital makes a significant impact on the asset allocation for individual investors' portfolios, thus developing models for incorporating it is an acute problem in the wealth management and personal financial planning field.

Given the importance of human capital for the asset allocation of individual investors' portfolios, this problem was already investigated by Bodie, Merton, Samuelson, Koo, Heaton, Lucas, Chen, Ibbotson, Davis. While we generally support the approach and conclusions of their research, there's one crucial point that requires a more careful examination. Davis[2] found that the return on human capital generally is uncorrelated with the stock market, and this assumption was also used by Bodie[3], Koo[4], and Heaton[5]. However, this assumption was challenged in our previous paper, and we found a substantial correlation between labor income and one- and two-year lagged stock market returns.

The purpose of this paper is to investigate ways of incorporating human capital and the fact it's correlated with stock market into the asset allocation process for individual investors. We start with

[1] Chen, P., Ibbotson, R., Milevsky, M., Zhu, K. (2006). Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance. *Financial Analysts Journal, Vol. 62, 1*, 97-109.

[2] Davis, S., Willen, P. (2000). Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice. *The National Bureau of Economic Research*. <http://www.nber.org/papers/w7905> (2016, February, 9).

[3] Bodie, Z., Merton, R., Samuelson, W. (1992). Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model. *Journal of Economic Dynamics and Control, No. 16*, 427-449.

[4] Koo, H.-K. (1995). Consumption and Portfolio Selection with Labor Income I: Evaluation of Human Capital. *Mathematical Finance, Vol. 8, 1*, 49-65.

[5] Heaton, J., Lucas, D. (1997). Market Frictions, Savings Behavior and Portfolio Choice. *Macroeconomic Dynamics, No. 1*, 76-101.

defining the human capital and presenting our findings on its correlation with stocks, after that we develop an idea of modeling this correlation mathematically, develop different models of wealth accumulation and present the results of Monte Carlo simulations. The paper is concluded by analysis of these results and their implications on portfolio management.

Defining the human capital

We define the human capital (HC) as the set of person's competences allowing her to earn labor income. As with any asset, the value of HC can be defined as the present value of all cash flows generated by it:

$$HC_t = \sum_{i=t}^{T} \frac{E\left[w_i\right]}{\left(1+r\right)^{i-t+1}} , \tag{1}$$

where HC_t is the value of HC at the beginning of year t , $E\left[w_t\right]$ is the expected salary or wage earned during the year t , T is the last year before retirement, r is the discount rate.

To estimate the expected salary we develop a simple career advancement model. At the moment we don't consider possible changes to one's human capital, which is definitely possible by reducing the working hours or improving one's skills. However, we do consider the fact the as people get experience their competences improve and their salaries increase. We also consider the fact that is a person is unemployed during the given year her expected salary in that year is zero. So, we model the expected salary as follows:

$$E\left[w_t\right] = w_0 k_t \left(1-P_u\right)\left(1+i\right)^t , \tag{2}$$

where w_0 is the average salary for fresh graduates as of the date of our research, k_t is a coefficient that accounts for increasing one's competences from being graduate to year t (we use the ratio of average salary of seasoned employees to average salary of fresh graduates) , P_u is the probability the person is unemployed (we use the average historical unemployment rate), i is the average inflation rate. Figure 1 shows the values of k_t and $w_0 k_t$ (which is the average *real* salary of an *employed* person in year t) we use in our research.

Figure 1. The modelled average real salary and salary increase coefficient

In our research we use the nominal cash flows, thus we need to account for inflation as appropriate. We do it by using assuming the long-term average inflation rate to be 2.25% (the average inflation as measured by CPI in 1872-2013). We discount each expected salary using the nominal risk-free rate, in our research we use the average rate on 10-years U.S. Treasuries as this risk-free rate. We also assume an average person starts employment at 25 years and retires at 65 years. The dynamics of HC for such a person given all the assumptions used in our research is shown on Figure 2.

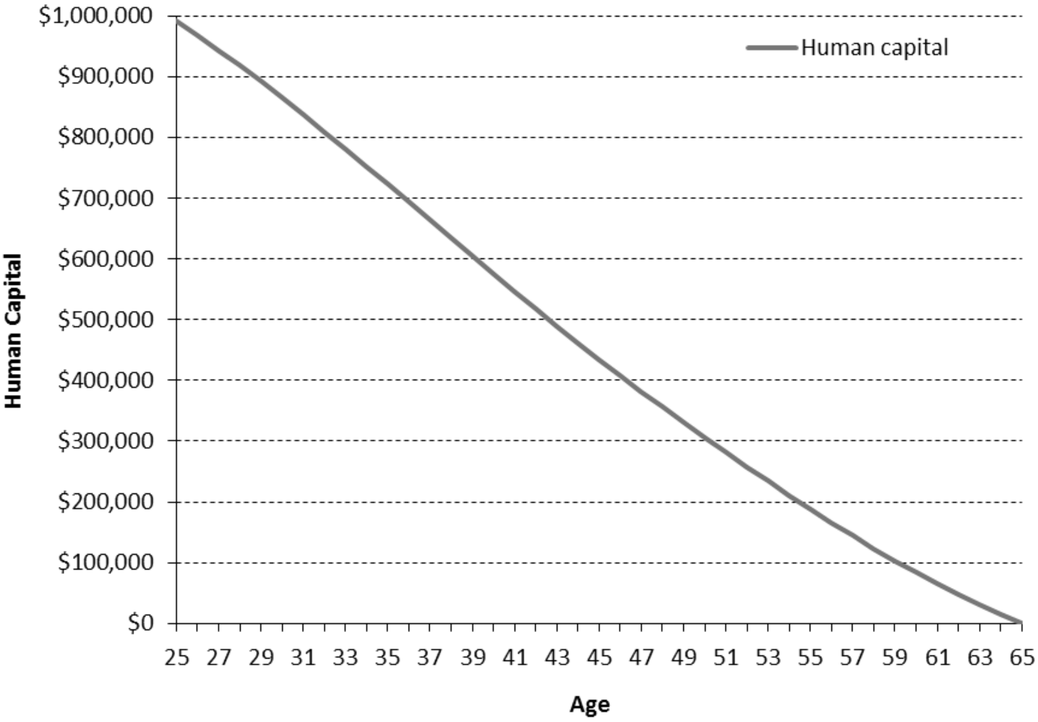

Figure 2. The dependence between human capital and the person's age

Correlation between labor income and the stock market returns

To find the correlation between labor income and the stock market returns we run a regression of the increase in wages in year *t* — $r_{w,t}$ — and stock market returns in years *t*, *t*–1, *t*–2 (denoted as r_t, r_{t-1}, r_{t-2}), and the inflation rate in year *t* (denoted as I_t). Supporting the position of Davis, we found no substantial correlation between $r_{w,t}$ and r_t, and the explanatory power of the same-year stock market returns is insignificant (R^2=0.0134). However, our findings, which are summarized in Table 1, show that the lagged stock market returns and inflation can explain a significant portion of the variance in labor income, so we can't ignore that dependence in our modeling. In this research we use the first model that accounts only for the one-year lagged stock market return. The correlation between it and the increase is wages is 0.76, and we will use this value in subsequent modeling.

Table 1.

Labor income regression on stock market returns and inflation rate

Model	$\mathrm{E}\left[r_{w,t}\right] = a_1 r_{t-1} + b$		$\mathrm{E}\left[r_{w,t}\right] = a_1 r_{t-1} + a_2 r_{t-2} + b$			$\mathrm{E}\left[r_{w,t}\right] = a_1 r_{t-1} + a_2 I_t + b$		
Variable	a_1	b	a_1	a_2	b	a_1	a_2	B
Estimate	0.1041	0.0253	0.0992	0.6903	0.0093	0.0999	0.0470	0.0218
(t-statistics)	(5.1209)	(5.9631)	5.1301	1.8378	0.9630	(5.4426)	(2.5432)	(5.2633)
R^2	0.5799		0.6956			0.6462		
Correlation	0.7615	n/a	0.7615	0.3592	n/a	0.7615	0.4063	n/a

To account for that correlation between the one-year lagged stock market returns and the wage increase in the current year, we need to generate a pair of random variables that have a joint normal distribution:

$$\begin{bmatrix} r_{t-1} \\ r_{w,t} \end{bmatrix} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}); \boldsymbol{\mu} = \begin{bmatrix} \overline{r}_s \\ \overline{r}_w \end{bmatrix}; \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_w \\ \rho\sigma_s\sigma_w & \sigma_w^2 \end{bmatrix} \quad (3)$$

where $r_{w,t}$ and r_t are respectively the wage increase and the stock market return in period t, $\overline{r}_s$ is the average stock market return, $\overline{r}_w$ is the average wage increase, σ_s is the standard deviation of the stock market returns, σ_w is the standard deviation of the labor income, ρ is the correlation between the one-year lagged year stock market returns and the current wage increase (which we set equal to 0.76).

The following formula can be used in order to generate a random variable $r_{w,t} = \Delta w_t / w_{t-1}$ that adheres to the joint normal distribution as specified in (3):

$$r_{w,t} = \Delta w_t / w_{t-1} = \overline{r}_w + \sigma_w z\sqrt{1-\rho^2} + \frac{\rho\sigma_w}{\sigma_s}\left(r_{t-1} - \overline{r}_s\right), \quad (4)$$

where $z \sim N(0,1)$ is the standard normally distributed random variable.

The asset allocation models

In this research we test three asset allocation models, all of which use the mean-variance optimization (MVO). The algorithm used for MVO is the Sharpe's algorithm[1], which is a quadratic programming algorithm that maximizes the following utility function:

$$U = E\left[r_p\right] - \frac{1}{r_t}\sigma_p^2,$$

where $E\left[r_p\right]$ is the expected return of the portfolio being optimized, σ_p^2 is its variance, and r_t is the risk tolerance.

Model 1. This is pure bonds-stocks model, without accounting for HC. We use AGG exchange-traded fund as the proxy for bonds and SPY exchange-traded fund as the proxy for stocks. We also assume the risk tolerance of the person decreases with the age, and use this model to calibrate it in such way that at the age of 25 the person is allocated 75% in the stocks and 25% in the bonds, and at the age of 65 the allocation is closest to 100% bonds.

Model 2. Now we add HC into the asset allocation. The amount of HC is assumed to be known and equal to the values calculated with formula (1). When running the MVO optimizer we model the fixed amount of HC by setting both its minimal and maximal allowed weights to the actual weight, so that the optimizer doesn't change it. We assume HC is uncorrelated with stocks, so it's actually a risk-free asset in many aspects. We also assume no correlation with bonds.

Model 3. This model is very similar to the previous one, but now we assume HC is correlated with stocks as described in the previous section. This model essentially depends on the volatility of the labor income. If the labor income had no volatility at all, it would have no correlation with the stock market, which isn't the case now. So it must be volatile. Unfortunately, we can't use the volatility of the mean labor income, because it's not the same as the average volatility of the labor income. Researchers haven't reached any definite conclusion on this topic either. Carroll[2] found that the average value for σ_w is 15%, while Chamberlain[3] estimates its median to be 10-11%, and Ranish[4] states that unconditional permanent log labor income standard deviation generally varies between 9% and 20%. Hence we analyzed several possible values for σ_w, and it turned out that the asset allocation critically depends on this parameter.

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[1] Sharpe, W. (1987). An Algorithm for Portfolio Improvement. *Advances in Mathematical Programming and Financial Planning*, 155-170.

[2] Carroll, C., Samwich, A. (1995). The nature of precautionary wealth. *The National Bureau of Economic Research*. < http://www.nber.org/papers/w5193> (2016, February, 9).

[3] Chamberlain, G., Hirano, K. (1999). Predictive distributions based on longitudinal earnings data. *Annales d'Economie et de Statistique, No. 55-56*, 211-242.

[4] Ranish, B. (2002). Why do Households with Risky Labor Income Take Greater Financial Risks? Job Market Paper, Harvard University.

Figure 3 shows the allocation to stocks for Models 1, Model 2, and Model 3. When we include HC into consideration, the allocation changes dramatically. For Models 2 and 3 before the age of 48-50 the suggested allocation would be 100% stocks, unlike the Model 1. Then the allocation rapidly changes to almost 100% bonds and at the age of 65 all models converge to the same allocation since HC decreases to zero.

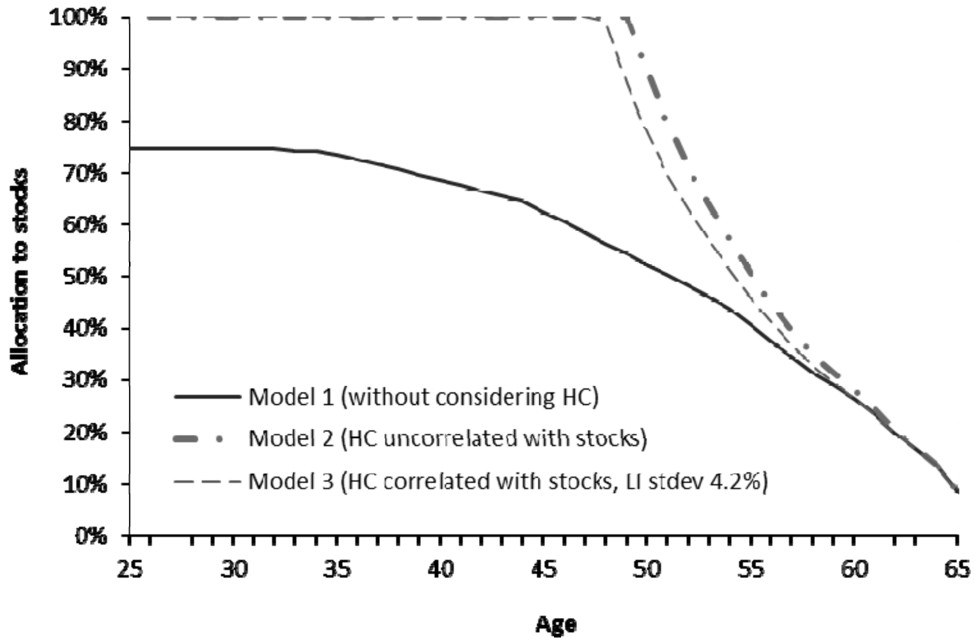

Figure 3. Allocation of financial assets to stocks for different models

The allocations to stocks generated with Model 3 under different assumptions regarding the value of σ_w are shown on Figure 4. When the standard deviation is close to zero, Model 2 and Model 3 generate very similar allocation, but as σ_w grows Model 3 suggests investing more into bonds at earlier age than Model 2. For standard deviations up to 18-19% the allocation strategy differs only with the age until which the person should invest 100% of the financial assets into stocks. But when σ_w gets to 20% (about the level of the standard deviation of stock market returns), the strategy changes dramatically. At your age, the person is suggested to be 100% invested into bonds, and start adding stocks to her portfolio after the age of 40.

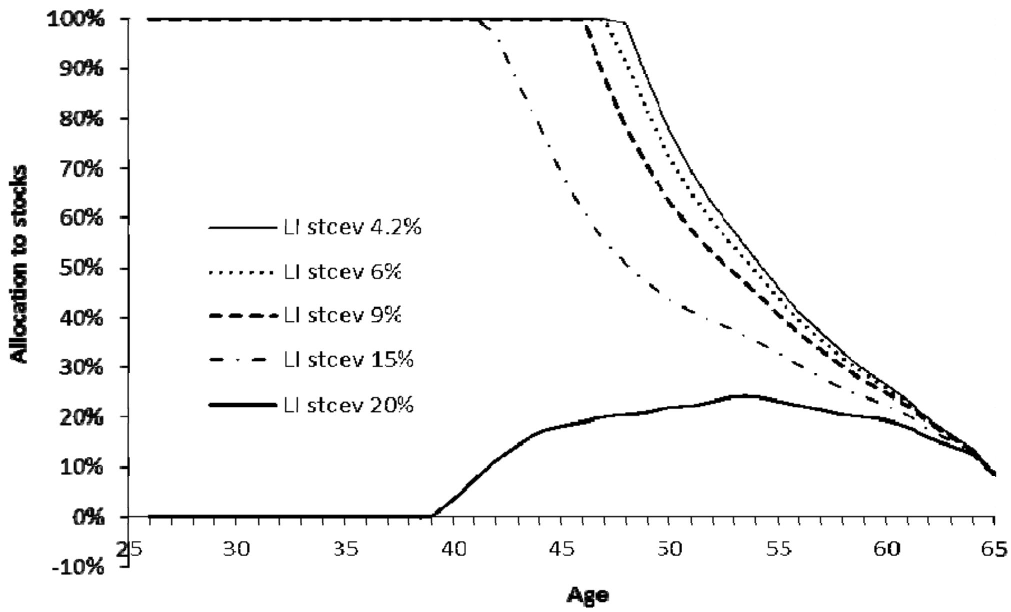

Figure 4. Allocation of financial assets to stocks for Model 3 depending on the standard deviation of labor income

The Monte-Carlo simulation of wealth accumulation

In order to check how three models presented above work in a more realistic environment, we conduct a series of Monte-Carlo simulations. Our target is to evaluate the total amount of wealth generated by a person during her working life. We assume that in a given year the person invests all the returns generated by the financial assets, as well as all labor income after taxes and living costs. In the simulation we use the standard U.S. personal income tax rate, as specified in Groppelli[1], and a simple living costs function

$$c_L = 7.2\left(w_{AT}\right)^{0.8}, \tag{5}$$

where c_L denotes the living costs, and w_{AT} is the after-tax salary.

For the simulation we did 10,000 runs, each of which is described below:

1. We start with a 25-year person without any financial assets and with initial salary \$24,982.
2. We run the MVO optimizer according to the settings of the specific model. The optimizer's outputs will be the asset allocation for the current year.
3. We generate the following random variables:
   - A normally distributed random variable N(9.3%, 20.5%) to represent the return on stocks
   - A normally distributed random variable N(4.5%, 5.4%) to represent the return on bonds
4. For a person aged 26 and above we calculate the wage increase during the current year as per (4):

$$w_t = w_{t-1}\left(1 + 4.3\% + N(0,1)\sigma_w\sqrt{1-0.76^2} + 0.78\frac{\sigma_w}{20.5\%}\left(r_{t-1} - 9.3\%\right)\right)$$

5. Using the asset allocation for the year, we calculate the return on the financial assets of the person and find their value at the end of the current year. If the resulting value is below zero, we suppose that the person loses all her investments and set the value of the financial assets to zero.
6. Then we add the after-tax salary less the living costs as per (5) to the value of the financial assets.
7. We repeat steps 2-6 until the person reaches 65 years. The total wealth accumulated by the person at 65 year equals to the value of the financial assets at that point, because the HC becomes zero.

Table 2.

Monte Carlo simulated personal wealth at the age of 65 years

Model	2	3	2	3	2	3
St. dev. of LI	6%	6%	15%	15%	20%	20%
Minimal wealth	55,772	5,152	788	236	106	1,087
Maximal wealth	10,168,315	8,086,623	29,563,340	17,875,397	36,764,998	80,578,086
Average wealth	1,135,349	1,084,342	1,239,361	1,081,377	1,375,256	1,071,232
St. dev. of wealth	695,587	752,833	1,403,798	1,450,890	2,083,748	2,245,005
5% percentile	361,215	274,193	58,848	21,368	20,281	9,651
10% percentile	461,247	362,407	124,105	57,250	51,890	24,849
Median wealth	975,122	905,590	802,243	602,863	684,428	412,366
90% percentile	2,009,186	2,021,401	2,850,748	2,617,789	3,440,704	2,626,398
95% percentile	2,434,217	2,509,055	3,900,065	3,759,687	4,921,844	4,249,836

In our research we conducted the Monte Carlo simulations for three different values of σ_w:: 6%, 15% and 20%. Their summary is provided in Table 2. As our simulation shows, Model 2 is superior to Model 3 for any level of labor income volatility, and especially it gives better outcomes as this level increases. The sample 5%, 10%, and 50% percentiles of the financial assets accumulated by a person as per our Monte Carlo simulation are shown on Figure 5, and there's little difference between Model 2 and Model 3 results at 6% LI standard deviation.

[1] Groppelli, A., Nikbakht, E. (2006). *Finance*, 5th ed., Barrons.

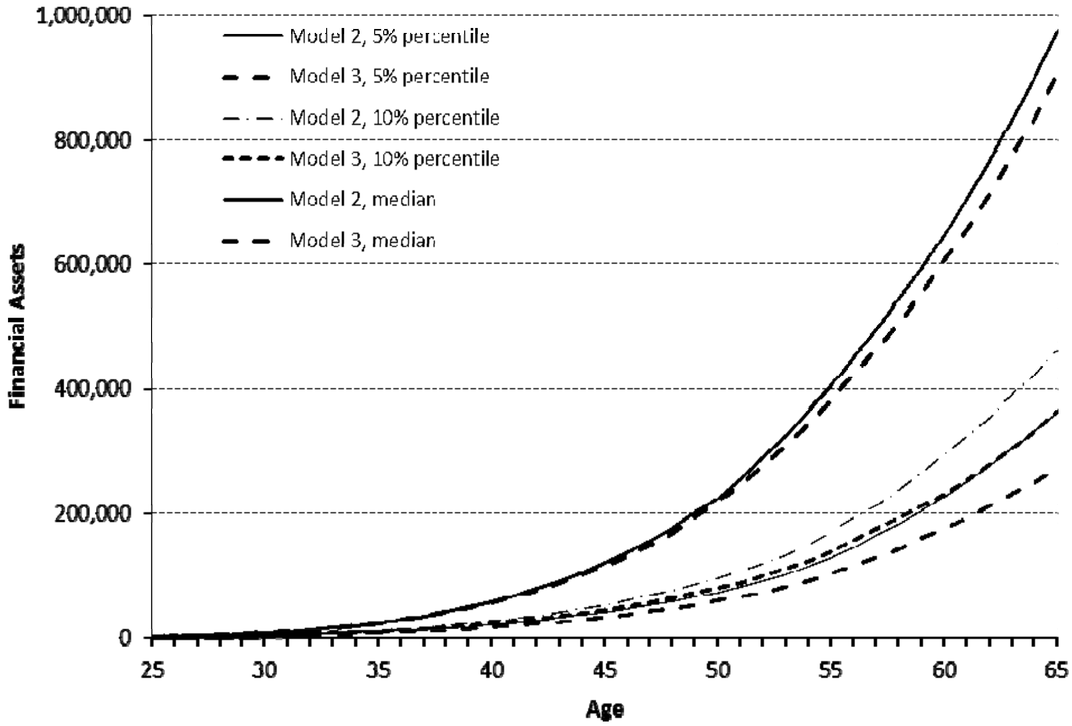

Figure 5. Percentiles for Monte-Carlo simulated wealth at LI standard deviation 6%

However, as LI standard deviation growth the difference between Model 2 and Model 3 becomes much more significant. For example, the same 5%, 10%, and 50% percentiles at 20% LI standard deviation are depicted on Figure 6, and we can notice how Model 3 results diverge from Model 2. Hence we conclude that on average Model 2 ensures a person ends her working life with a greater wealth. So, contrary to our initial hypothesis, including the correlation of labor income and stock market returns seems to have little sense in the asset allocation.

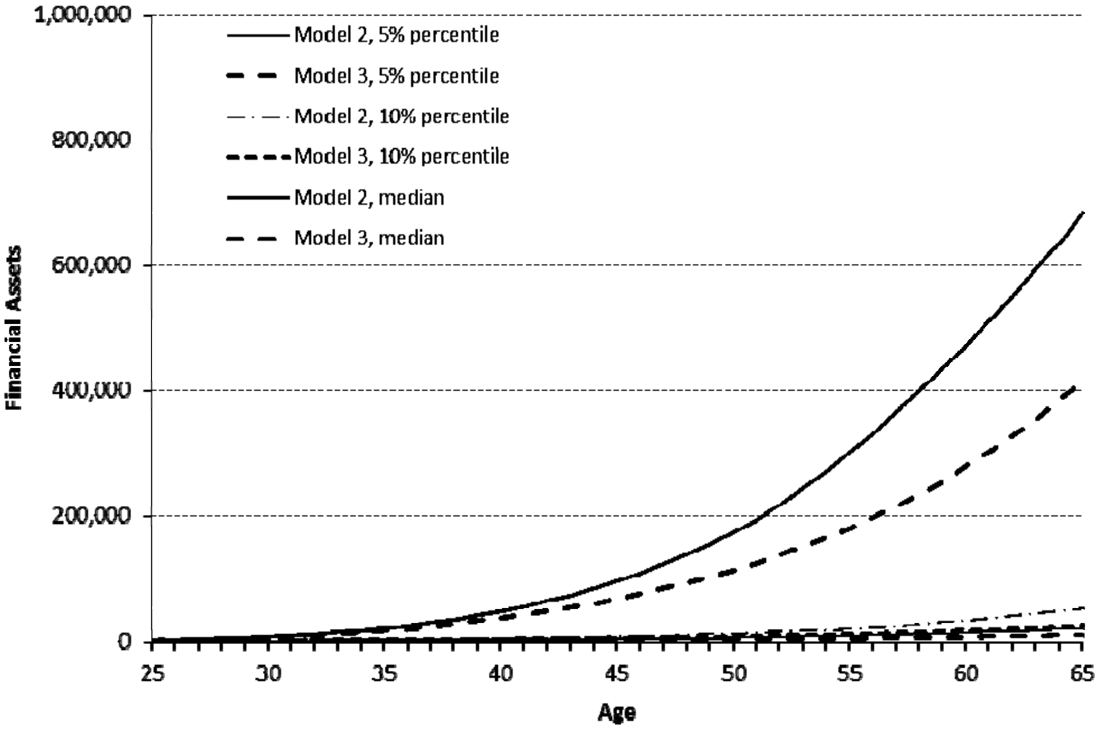

Figure 6. Percentiles for Monte-Carlo simulated wealth at LI standard deviation 20%

Conclusions

Incorporating human capital into the asset allocation leads to a striking difference in allocations comparing to those that are generated by considering only the financial assets. Adding the correlation between the return on human capital and the stock market return leads to a subtler difference. It becomes significant only when the standard deviation of labor income is comparable to that of the stock market returns. Actually, if this standard deviation is small it doesn't really matter if the return on human capital in correlated with the stock market return or not.

We failed to prove our hypothesis that including the correlation between the return on human capital and the stock market return into portfolio optimization leads to a superior asset allocation. The Monte Carlo simulations show that incorporating this correlation is such a straightforward way as we did leads to inferior results. It's simpler and safer at the moment to assume that the return on human capital is uncorrelated with the stock market return. Nevertheless, a more careful incorporating the correlation between labor income and one-year lagged stock market return would be a perspective direction of further research.

References

1. Bodie, Z., Merton, R., Samuelson, W. (1992). Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model. *Journal of Economic Dynamics and Control, No. 16*, 427-449.
2. Carroll, C., Samwich, A. (1995). The nature of precautionary wealth. *The National Bureau of Economic Research*. <http://www.nber.org/papers/w5193> (2016, February, 9).
3. Chamberlain, G., Hirano, K. (1999). Predictive distributions based on longitudinal earnings data. *Annales d'Economie et de Statistique, No. 55-56*, 211-242.
4. Chen, P., Ibbotson, R., Milevsky, M., Zhu, K. (2006). Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance. *Financial Analysts Journal, Vol. 62, 1*, 97-109.
5. Davis, S., Willen, P. (2000). Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice. *The National Bureau of Economic Research*. <http://www.nber.org/papers/w7905> (2016, February, 9).
6. Groppelli, A., Nikbakht, E. (2006). *Finance*, 5th ed., Barrons.
7. Heaton, J., Lucas, D. (1997). Market Frictions, Savings Behavior and Portfolio Choice. *Macroeconomic Dynamics, No. 1*, 76-101.
8. Koo, H.-K. (1995). Consumption and Portfolio Selection with Labor Income I: Evaluation of Human Capital. *Mathematical Finance, Vol. 8, 1*, 49-65.
9. Ranish, B. (2002). Why do Households with Risky Labor Income Take Greater Financial Risks? Job Market Paper, Harvard University.
10. Sharpe, W. (1987). An Algorithm for Portfolio Improvement. *Advances in Mathematical Programming and Financial Planning*, 155-170.