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MODEL OF ENTERPRISE STRATEGIC POTENTIAL ESTIMATION IN THE FORM OF DISCRETE-TIME OPTIMAL CONTROL PROBLEM

Due to the optimization essence of the company's potential its evaluation requires to determine the optimal parameters to control enterprise's activity. To determine the optimal control of dynamic systems, both analytical and simulation models are used. This paper demonstrates the possibility of evaluating the enterprise's strategic potential by means of discrete problem of optimal control. In this work, the discrete-time optimal control problem for enterprise strategic potential estimation is build taking into account a set of variants of managerial decisions in the field of marketing. On the conditional example, the solution of the constructed problem is demonstrated with the help of the method of dynamic programming.
Keywords: result estimation of enterprise strategic potential, model of potential activity of enterprise, discrete-time optimal control problem, dynamic programming.

Statement of the problem. The concept of the strategic potential of enterprises is a popular subject of modern economic research. That is quite reasonable, as the strategic potential of enterprises characterizes long-term opportunities for their future functioning and development; it is an important factor in the progress of the economy as a whole. Therefore, one of the priority tasks of management at both microeconomics and macroeconomics levels is the implementation of well-grounded management of the strategic potential of enterprises.

Management of the strategic potential of a company is based on its estimation. Today, subjective methods are traditionally used to estimate a company's strategic potential; such methods are based on the experience and intuition of experts. For example, Ye. V. Lagunova[1] uses the VRIO-analysis proposed by J. Barney[2] to estimate a company's strategic potential. T. M. Kibuk[3] uses the method of additive convolution of partial indicators of the potential's functional components, in which estimates of partial indices and weighting factors are set expertly. V. P. Vasileva in her dissertation[4] considers the following basic methods to estimate the strategic potential of an enterprise: SWOT-analysis, benchmarking, stakeholder analysis, cost chain analysis (according to M. Porter), skills analysis, flexibility analysis, GAP-analysis, human resource analysis.

Widespread application of expert methods for assessing the strategic potential of enterprises is stipulated by the insufficient development of objective methods for mathematical modeling of potential's assessment.

In order to develop objective methods of mathematical modeling of enterprise potential estimation, the author proposes a general model of enterprise potential estimation in the notation of a discrete problem of optimal control[5]. This model looks as follows:

$$\sum_{\tau=1}^{N} r(x_\tau, \tilde{s}_\tau, s_\tau, z_\tau) \to \max_{(s_\tau)}, \qquad (1)$$

[1] Лагунова, Е.В. (2007). Стратегический потенциал компании и его оценка. *Проблемы управления*, *6*, 40–44.
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$$s_\tau \in S^{set}(x_\tau, \tilde{s}_\tau, z_\tau), \tag{2}$$

$$x_{\tau+1} = x_\tau + d(x_\tau, \tilde{s}_\tau, s_\tau, z_\tau), \; x_1 = \tilde{x}, \; \tau = \overline{1..N}, \tag{3}$$

where: τ – the number of the time interval within the planning period; $r_\tau = r(x_\tau, \tilde{s}_\tau, s_\tau, z_\tau)$ – profit of the enterprise within the interval τ; x_τ – vector of characteristics of enterprise resources in an interval τ; $\tilde{x}$ – the given vector of characteristics of enterprise resources at the beginning of the planning period; S_τ – vector of controlled parameters of enterprise activity in an interval τ; $S^{set}(x_\tau, \tilde{s}_\tau, z_\tau)$ – limits imposed on a set of permissible vector values S_τ; $\tilde{S}_\tau$ – vector of the specified parameters of the enterprise activity in the interval τ; z_τ – vector of the parameters of the environment in the interval τ; $d_\tau = d(x_\tau, \tilde{s}_\tau, s_\tau, z_\tau)$ – vector of growth of values of resources' characteristics.

The given model is general and requires the specification and taking into account the features of a certain level of potential of an enterprise (strategic, tactical or operational). The stated fact predetermined the purpose of the study.

The purpose of the study. The purpose of this study is to construct a model for evaluating the strategic potential of an enterprise in the form of a discrete problem of optimal control taking into account the existence of a plurality of options for managerial decisions in the field of marketing.

Presentation of the basic material. According to the three-level concept of potential[1], the strategic potential of an enterprise is the ability of an enterprise to realize its reproduction and satisfy the interests of stakeholders by using a mechanism of strategic adaptation in the long run. The estimation of the strategic potential of an enterprise is the maximum value of the performance indicators that can be achieved by an enterprise during the long-term planning period (under different environmental conditions). The model for assessing the strategic potential of an enterprise should have the following features compared to models for assessing tactical and operational levels of capacity: 1. This model should have a generalized, aggregated character in order to give a general picture of the dynamics of an enterprise's capabilities in a high level of uncertainty exogenous information. 2. Options for business strategy and characteristics of projects for an enterprise's development should be controlled variables of a model. 3. The model's limitations must be specified by characteristics of the strategic resources of an enterprise, that is, those that are the most inert and determining the capacity of an enterprise over the long term (such as equipment fleet and customer relationship). 4. The model should take into account the dynamics of strategic resources of an enterprise.

We will build a model for estimation the strategic potential of an enterprise, in which will take into account the many options for managerial decisions on the number of advertising projects. In this paper, we will consider a simplified version of the model, which will take into account only one result indicator of an enterprise – the indicator of net profit – as a characteristic of the growth of economic capital of an enterprise. We also consider the given dynamics of production capacities of an enterprise.

Taking into account the above, the model of strategic potential estimation of an enterprise under the controlled number of advertising projects and the given dynamics of production capacities will look as follows:

a) target function

$$\sum_{\tau=1}^{N} r_\tau = \sum_{\tau=1}^{N} (1 - z^{\varphi n1}) \cdot [\sum_i a_i^v \cdot s_{i\tau}^y - a^c - z^{\varphi\mu a} \cdot s_\tau^{\mu a})] \to \max_{(s_{i\tau}^y),(s_\tau^{\mu a})}; \tag{4}$$

b) limitation

$$\sum_i s_{i\tau}^y \cdot x_{i\tau}^{\pi ea} \le f^\pi, \; \forall \tau \;, \tag{5}$$

[1] Заруба, В.Я., Потрашкова, Л.В. (2010). Системный подход к анализу потенциала предприятия. *Вісник Східноукраїнського національного університету ім. В. Даля*, *8(150),* 59–62.

$$s_{i\tau}^{y} \le z_i^{\mu} + x_{i\tau}^{\mu d} \ , \quad \forall i,\tau \ , \tag{6}$$

$$r_{\tau} \ge 0 \ , \quad \forall \tau \ , \tag{7}$$

$$s_{i\tau}^{y} \ge 0 \ , \quad \forall i,\tau \quad , \quad s_{\tau}^{\mu a} \in \{0,1\} \ ; \tag{8}$$

c) correlation that specify the dynamics of equipment

$$N_{\tau+1}^{\pi e} = N_{\tau}^{\pi e} + N_{\tau}^{\pi e+}, \quad N_1^{\pi e} = \tilde{N}^{\pi e} \ , \tag{9}$$

$$\sum_{\alpha=1}^{N_{\tau}^{\pi e+}} z_{N_{\tau}^{\pi e}+\alpha}^{\pi e \varphi q} \le r_{\tau} \ , \tag{10}$$

$$I_{\tau}^{\pi e} = \{ q \mid 1 \le q \le N_{\tau}^{\pi e} \} \ , \tag{11}$$

$$x_{i\tau}^{\pi ea} = 1 / \sum_{q \in I_{\tau}^{\pi e}} z_{qi}^{\pi eaq/} \ ; \tag{12}$$

d) correlation that specify the dynamics of additional demand

$$x_{\tau+1}^{\mu a} = s_{\tau}^{\mu a} \ , \quad x_1^{\mu a} = 0 \ , \tag{13}$$

$$x_{i\tau}^{\mu d} = z_i^{\mu a1} \cdot s_{\tau}^{\mu a} + z_i^{\mu a2} \cdot x_{\tau}^{\mu a} \ ; \tag{14}$$

$$i \in \tilde{S}^{w} \ , \quad \tau = \overline{1..N} \ ; \tag{15}$$

where: r_{τ} – net profit of an enterprise in the interval of time τ;

N – the number of time intervals within the planning period;

controlled variables:

$s_{i\tau}^{y}$ – volume of production of *i*-type products at intervals of time τ;

$s_{\tau}^{\mu a}$ – the number of marketing projects in the interval τ, $s_{\tau}^{\mu a} \in \{0,1\}$;

parameters set by business strategy:

$N_{\tau}^{\pi e+}$ – the specified number of units of equipment put into operation at an enterprise on an interval τ; $N^{\pi e+} = \sum_{\tau=1}^{N} N_{\tau}^{\pi e+}$;

$\tilde{S}^{w}$ – a number of types of products produced by an enterprise in the planned period;

a_i^{v} – the specific value "income minus variable costs" per unit of *i*-type products;

a^c – constant costs of an enterprise at one interval of time (without taking into account advertising costs);

parameters of the environment:

$z^{\varphi n1}$ – tax rate on profit;

$z^{\varphi\mu a}$ – the cost of an advertising project;

f^{π} – time work fund at one time interval;

$z_{qi}^{\pi eaq/}$ – productivity of q-type unit of equipment in production of i-type products ($q = \overline{1, N_1^{\pi e} + N^{\pi e+}}$);

$z_q^{\pi e\varphi q}$ – cost of q-type unit of equipment ($q = \overline{N_1^{\pi e} + 1, N_1^{\pi e} + N^{\pi e+}}$);

z_i^{μ} – "guaranteed" volume of demand for i-type products (at one interval of time), which exists regardless of the presence of advertising;

$z_i^{\mu a1}$ – the coefficient that characterizes the dependence of demand for i-type products on advertising of the current period;

$z_i^{\mu a2}$ – the coefficient that characterizes the dependence of demand for i-type products on advertising of the past period;

endogenous variables that describe the characteristics of an enterprise's resources and their dynamics:

$x_\tau^{\mu a}$ – the number of last year's advertising projects that affect the level of demand at the interval τ ($x_\tau^{\mu a} \in \{0,1\}$);

$x_{i\tau}^{\mu d}$ – additional demand for i-type products at τ interval, formed as a result of the implementation of advertising projects in the planning period;

$x_{i\tau}^{\pi ea}$ – time consuming equipment complex for production of a unit of i-type product at τ interval;

$N_\tau^{\pi e}$ – the number of equipment units at the enterprise at the beginning of the interval τ; $N_1^{\pi e} = \tilde{N}^{\pi e}$;

$I_\tau^{\pi e}$ – Set of numbers of all units of production equipment at the enterprise at the beginning of the τ interval.

Let's consider the application of the dynamic programming method to solve the above model on a conditional example.

Let the strategic planning period cover 4 intervals (years), $N = 4$. During the first three years, the company will annually purchase and put into operation a new unit of equipment, increasing its production capacity ($N_\tau^{\pi e+}$=1 for $\tau = \overline{1,3}$). And in each of these three years the company's management has a choice: whether or not to implement a certain large-scale advertising project aimed at increasing demand for products, $s_\tau^{\mu a} \in \{0,1\}$. By criterion of total net profit there is a certain optimal plan $(s_\tau^{\mu aOPT}) = (s_1^{\mu aOPT} \quad s_2^{\mu aOPT} \quad s_3^{\mu aOPT})$. The value of total profit in the planned period at the optimal plan $(s_{\tau}^{\mu aOPT})$ will characterize the size of the strategic potential of the enterprise in the given environment.

Let the model parameters have the following values: $\tilde{S}^{w}=\{1,2\};$ $a_1^v=1400$; $a_2^v=1500$; $a^c=10000$; $z^{\varphi n1}=0{,}18,$ якщо $r_\tau \geq 0$; $z^{\varphi \mu a}=30000$; $f^{\pi}=2000$; $\tilde{N}^{\pi e}=3$; $\tilde{I}^{\pi e}=\{1,2,3\}$; $z_{qi}^{\pi eaq/}=0{,}05$ for $i=\overline{1,2}$, $q=\overline{1,6}$; $z_1^{\mu}=200$; $z_2^{\mu}=100$; $z_1^{\mu a1}=z_2^{\mu a1}=10$; $z_1^{\mu a2}=z_2^{\mu a2}=5$.

The state of the enterprise resource system at the beginning of each year is described by the number of operating units of equipment and the number of advertising projects carried out in the previous interval (more precisely, the level of demand caused by the implementation of these advertising projects): $x_\tau=(x_3^{\pi e},x_3^{\mu a})$.

To solve the above conditional example, we apply the principle of Bellman's optimality. At any state of the system, at an arbitrary step, it is necessary to choose the control so that profit at this step plus the optimal total profit at all subsequent steps is maximal.

For the problem under consideration, the Bellman equation has the following form:

$$F_\tau(x_\tau)=\max_{s_\tau^{\mu a}}[r_\tau(x_\tau,s_\tau^{\mu a})+F_{\tau+1}(x_{\tau+1}(x_\tau,s_\tau^{\mu a}))],$$

where: $F_\tau(x_\tau)$ – The maximum total net profit that an enterprise receives when it moves from x_τ state to the final state x_N under optimum control;

$r_\tau(x_\tau,s_\tau^{\mu a})$ – net profit of the enterprise in the interval of time τ, which depends on the state of the enterprise's resource system x_τ and selected management $s_\tau^{\mu a}$, $\tau=\overline{1,N}$;

$s_\tau^{\mu a}$ – management that transfers the system from the state x_τ to $x_{\tau+1}$ state (management $s_\tau^{\mu a*}$, at which the value is reached $\max\limits_{s_\tau^{\mu a}}[r_\tau(x_\tau,s_\tau^{\mu a})+F_{\tau+1}(x_{\tau+1}(x_\tau,s_\tau^{\mu a}))]$, is called conventionally optimal).

As it follows from the Bellman principle of optimality, an optimal management strategy can be obtained if first we find the optimal management strategy at the last step, then in the last two steps, then in the last three steps, and so on, until the first step.

As at the last interval of time $\tau=N=4$ marketing projects are not implemented, so $F_4(x_4)=r_4(x_4^{\pi e},x_4^{\mu a},s_4^{\mu a})=r_4(x_4^{\pi e},x_4^{\mu a},0)$. So calculations should be started with finding optimal control on the interval $\tau=3$.

According to the Bellman equation, as $s_\tau^{\mu a}\in\{0,1\}$, so:

$$F_3(x_3)=\max_{s_3^{\mu a}}[r_3(x_3^{\pi e},x_3^{\mu a},s_3^{\mu a})+F_4(x_4(x_3^{\pi e},x_3^{\mu a},s_3^{\mu a}))]=$$

$$=\max\begin{cases} r_3(x_3^{\pi e},x_3^{\mu a},1)+r_4(x_4^{\pi e}(x_3^{\pi e},x_3^{\mu a},1),x_4^{\mu a}(x_3^{\pi e},x_3^{\mu a},1),0) \\ r_3(x_3^{\pi e},x_3^{\mu a},0)+r_4(x_4^{\pi e}(x_3^{\pi e},x_3^{\mu a},0),x_4^{\mu a}(x_3^{\pi e},x_3^{\mu a},0),0) \end{cases}.$$

At the beginning of the 3rd interval of the planning period, such variants of the state of the resource system of an enterprise are possible $x_3 = (x_3^{\pi e} \quad x_3^{\mu a})$:

1. $x_3^{\pi e} = 2, x_3^{\mu a} = 0$

In this condition of the system the next variants of the values of future characteristics of the system are possible:

$$\text{if } s_3^{\mu a} = 1, \text{ that: } r_3(x_3^{\pi e}, x_3^{\mu a}, 1) = r_3(2,0,1) = 343580,$$
$$x_4^{\pi e} = 3, x_4^{\mu a} = 1;\ r_4(x_4^{\pi e}, x_4^{\mu a}, 0) = r_4(3,1,0) = 356290;$$
$$\text{if } s_3^{\mu a} = 0, \text{ that: } r_3(x_3^{\pi e}, x_3^{\mu a}, 0) = r_3(2,0,0) = 344400,$$
$$x_4^{\pi e} = 3, x_4^{\mu a} = 0;\ r_4(x_4^{\pi e}, x_4^{\mu a}, 0) = r_4(3,0,0) = 344400.$$

Then $F_3(2,0) = max \begin{cases} r_3(x_3^{\pi e}, x_3^{\mu a}, 1) + r_4(x_4^{\pi e}, x_4^{\mu a}, 0) \\ r_3(x_3^{\pi e}, x_3^{\mu a}, 0) + r_4(x_4^{\pi e}, x_4^{\mu a}, 0) \end{cases} = 699870.$

Hence, $s_3^{\mu a*} = 1$ (conventionally optimal control at the interval $\tau =$ 3).

2. $x_3^{\pi e} = 2, x_3^{\mu a} = 1$

In this condition of the system the next variants of the values of future characteristics of the system are possible:

$$\text{if } s_3^{\mu a} = 1, \text{ that: } r_3(x_3^{\pi e}, x_3^{\mu a}, 1) = r_3(2,1,1) = 355470,$$
$$x_4^{\pi e} = 3, x_4^{\mu a} = 1;\ r_4(x_4^{\pi e}, x_4^{\mu a}, 0) = r_4(3,1,0) = 356290;$$
$$\text{if } s_3^{\mu a} = 0, \text{ that: } r_3(x_3^{\pi e}, x_3^{\mu a}, 0) = r_3(2,1,0) = 356290,$$
$$x_4^{\pi e} = 3, x_4^{\mu a} = 0;\ r_4(x_4^{\pi e}, x_4^{\mu a}, 0) = r_4(3,0,0) = 344400.$$

Then $F_3(2,1) = max \begin{cases} r_3(1,1,1) + r_4(1,2,0) \\ r_3(1,1,0) + r_4(2,1,0) \end{cases} = 711760;\ s_3^{\mu a*} = 1.$

At the beginning of the 2nd year of the planning period, are possible the next options for the state of an enterprise's resource system $x_2 = (x_2^{\pi e} \quad x_2^{\mu a})$:

1. $x_2^{\pi e} = 1, x_2^{\mu a} = 0$

In this condition of the system the next variants of the values of future characteristics of the system are possible:

$$\text{if } s_2^{\mu a} = 1, \text{ that: } r_2(x_2^{\pi e}, x_2^{\mu a}, 1) = r_2(1,0,1) = 343580,$$
$$x_3^{\pi e} = 2, x_3^{\mu a} = 1;\ F_3(2,1) = 711760;$$

if $s_2^{\mu a}=0$, that: $r_2(x_2^{\pi e},x_2^{\mu a},0)=r_2(1,0,0)=344400$,

$x_3^{\pi e}=2, x_3^{\mu a}=0$; $F_3(2,0)=699870$.

Then $F_2(1,0)=max\begin{cases} r_2(1,0,1)+F_3(2,1) \\ r_2(1,0,0)+F_3(2,0) \end{cases}$ =1055340; $s_2^{\mu a*}=1$;

2. $x_2^{\pi e}=1, x_2^{\mu a}=1$

In this condition of the system the next variants of the values of future characteristics of the system are possible:

if $s_2^{\mu a}=1$, that: $r_2(x_2^{\pi e},x_2^{\mu a},1)=r_2(1,1,1)=355470$,

$x_3^{\pi e}=2, x_3^{\mu a}=1$; $F_3(2,1)=711760$;

if $s_2^{\mu a}=0$, that: $r_2(x_2^{\pi e},x_2^{\mu a},0)=r_2(1,1,0)=356290$,

$x_3^{\pi e}=2, x_3^{\mu a}=0$; $F_3(2,0)=699870$.

Then $F_2(1,1)=max\begin{cases} r_2(1,1,1)+F_3(2,1) \\ r_2(1,1,0)+F_3(2,0) \end{cases}=1067230$; $s_2^{\mu a*}=1$.

At the beginning of the first year of the planning period, the resource system of the enterprise is in a condition $x_1=(0\quad 0)$. In this condition of the system the next variants of the values of future characteristics of the system are possible:

if $s_1^{\mu a}=1$, that: $r_1(x_1^{\pi e},x_1^{\mu a},1)=r_1(0,0,1)=320620$,

$x_2^{\pi e}=1, x_2^{\mu a}=1$; $F_2(1,1)=1067230$;

if $s_1^{\mu a}=0$, that: $r_1(x_1^{\pi e},x_1^{\mu a},0)=r_1(0,0,0)=344400$,

$x_2^{\pi e}=1, x_2^{\mu a}=0$; $F_2(1,0)=1055340$.

Then $F_1(0,0)=max\begin{cases} r_1(0,0,1)+F_2(1,1) \\ r_1(0,0,0)+F_2(1,0) \end{cases}=1399740$; $s_1^{\mu a*}=0$.

Let's consider the obtained results in reverse order. As $x_1=(0\quad 0)$ i $s_1^{\mu a*}=0$, so $x_2=(1\quad 0)$. For the variant $x_2=(1\quad 0)$ conventionally optimal control has been determined $s_2^{\mu a*}=1$. Then $x_3=(2\quad 1)$. For the variant $x_3=(2\quad 1)$ conventionally optimal control has been determined $s_3^{\mu a*}=1$. Then $x_4=(3\quad 1)\cdot$ Problem solving: $s^{\mu aOPT}=(0\quad 1\quad 1)$. This task solution corresponds to the value of the total net profit of the enterprise in the planning period $r^{sum*}=1399740$. It is this value that characterizes the magnitude of the strategic potential of the enterprise in the given parameters of the environment.

Conclusions. Due to the optimization essence of the enterprise's potential, its evaluation involves determining the optimal enterprise management parameters. As you know, to determine the optimal control parameters of dynamic systems, both analytical and simulation models are used. This paper demonstrates the possibility of evaluating the strategic potential of an enterprise based on the construction of a discrete problem of optimal control.

Further developments in the direction of research may be aimed at developing a simulation approach to assess the strategic potential of an enterprise.

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